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LAMINAR FILM CONDENSATION IN A TUBE WITH UPWARD VAPOR FLOW

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Abstract—The Nusselt solution for laminar film condensation on a vertical plate with upward vapor flow is reviewed, with comments on the nature of the condensate flow. The same laminar film condensation problem is considered for the case of upward vapor flow in the inside of a vertical tube, in which case the decrease in the steam flow rate due to the condensation produces a reduction in the shear stress which supports the condensate layer. A numerical solution of the problem is then required and results are given for specific cases to illustrate the method of the calculation and to indicate the nature of the condensation process in the vertical tube. These cases involve both upward and downward flow of the condensate.

NOMENCLATURE

c_f, friction coefficient, $(2\tau_0)/(\rho u^2)$;
D, tube inside diameter;
F, friction factor augmentation factor;
g, gravitational acceleration;
G, $\left(1 + \dfrac{1}{\rho_L g}\dfrac{dP}{dz}\right)$, equation (2);
k, thermal conductivity;
$\dot{m}$, mass rate of flow;
$\dot{m}''$, mass flux;
N, dimensionless quantity, N_1, equation (6), N_2, equation (3a);
P, pressure;
T, temperature;
u, velocity in the z direction, $\bar{u} = u/(g\nu)^{1/3}$;
y, distance normal to the wall, $\bar{y} = y/(\nu^2/g)^{1/3}$;
z, distance along the plate or the tube normal from the bottom edge.

Greek symbols
μ, dynamic viscosity;
ν, kinematic viscosity;
τ, shear;
δ, liquid film thickness;
λ, latent heat of vaporization;
Γ, liquid mass flow rate, per unit film width;
ρ, density.

Subscripts
L, liquid;
v, vapor;
δ, edge of liquid film;
c, condensation.

Superscript
$\bar{\ }$, dimensionless quantity in terms of $(\nu^2/g)^{1/3}$ as a distance measure of $(g\nu)^{1/3}$ as a velocity measure.

1. INTRODUCTION

THIS report is a consideration of the problem of the condensation of a pure saturated vapor flowing upward in a vertical tube, for constant liquid and vapor properties, under conditions for which the liquid flow remains laminar. This problem is associated with that of Nusselt [1] who considered the problem of condensation on a plate of finite height with a vapor velocity past the plate surface; in this problem, depending on the plate height, the liquid can flow either upward or downward on the plate. The results of the Nusselt solution are given by Jakob [2] and by Kutateladze [3]. This solution is not applicable to the pipe situation because in the pipe the vapor velocity diminishes as the condensation proceeds.

Rohsenow (4) has presented a method for the calculation of the condensation in a tube which assumes co-current annular flow of the vapor and of the liquid. On this basis the pressure drop is calculated from the correlation of Lockhart and Martinelli, so that the pressure gradient is found in terms of the local quality. For the heat transfer there is used a correlation specification of the heat transfer coefficient in terms of the two phase flow parameter, X_{tt}; involving the quality and the properties. Then, by assuming increments in the quality, the heat transfer out of the pipe wall in the increment is specified and, with the heat transfer coefficient determined, the length increment is evaluated. This solution then gives the quality and the pressure as function of the distance from the location at which condensation begins.

The solution given here is more closely associated with the solution of Nusselt, in that a laminar liquid flow of the Nusselt type is specified. Thus, there is in the pipe an annular flow. This is specified in terms of a friction coefficient at the interface, the coefficient depending on the gas flow. In the vertical pipe, as in the Nusselt solution for the vertical plate, there is a restricted regime for totally upward condensate flow, though it appears that there is an undefined set of circumstances for which the condensate can flow both upward and downward. In this latter case the postulates of the Rohsenow method based on co-current flow of the liquid and the vapor would probably be unsuitable.

In respect to the present problem, and for an orientation to it, some further discussion of the Nusselt problem is included.

2. THE MOTION OF THE LIQUID

The motion of the liquid is specified by neglecting the acceleration terms in the momentum equations, and assuming the liquid layer to be so thin relative to the radius of the pipe that the curvature of the layer can be neglected. Then for gravity in the downward direction

$$0 = \mu_L \frac{d^2u}{dy^2} - \frac{dP}{dz} - \rho_L g, \tag{1}$$

$$\frac{\mu_L}{\rho_L g}\frac{d^2u}{dy^2} = \left(1 + \frac{1}{\rho_L g}\frac{dP}{dz}\right) = G. \tag{2}$$

Integration yields for the velocity distribution in $0 < y < \delta$, where δ is the layer thickness

$$\frac{\mu_L}{\rho_L g}u = G\left(\frac{y^2}{2} - \delta y\right) + \frac{\tau_\delta}{\rho_L g}y \tag{3}$$

and from this velocity distribution the liquid Reynolds number is obtained as

$$\frac{\mu_L^2}{\rho_L^2 g}\int_0^\delta \frac{u\,dy}{\nu_L} = -G\frac{\delta^3}{3} + \frac{\tau_\delta}{\rho_L g}\frac{\delta^2}{2}. \tag{4}$$

Non-dimensional variables are specified as

$$\bar\delta = \delta\left(\frac{g}{\nu_L^2}\right)^{1/3} \quad \bar y = y\left(\frac{g}{\nu_L^2}\right)^{1/3} \quad \bar u = u/(\nu_L g)^{1/3}.$$

With these variables the velocity distribution specified by equation (3) becomes

$$\bar u = G\left(\frac{\bar y^2}{2} - \bar\delta\bar y\right) + \frac{\tau_\delta}{\rho_L g}\left(\frac{g}{\nu_L^2}\right)^{1/3}\bar y \tag{3a}$$

and in particular, the liquid velocity at the edge of the condensate layer is

$$\bar u_\delta = -G\frac{\bar\delta^2}{2} + \frac{\tau_\delta}{\rho_L g}\left(\frac{g}{\nu_L^2}\right)^{1/3}\bar\delta. \tag{3b}$$

The non-dimensional group that is the coefficient of $\bar\delta$ in the last term is specified as N_2; it is a function of the shear, τ_δ, at the liquid–vapor interface.

The Reynolds number, as given by equation (4), becomes, for $G = 1$

$$\frac{1}{\nu_L}\int_0^\delta u\,dy = \frac{\Gamma}{\mu_L} = N_2\frac{\bar\delta^2}{2} - \frac{\bar\delta^3}{3}. \tag{4a}$$

A local mass balance specifies the change in the Reynolds number in terms of the condensation flux $\dot m_c''$ ($kg\,s^{-1}\,m^{-2}$)

$$\mu_L\frac{d(\Gamma/\mu_L)}{dz} = \dot m_c''.$$

If the convective terms in the energy equation are neglected, that is, if the contribution of liquid subcooling to the energy flux in the layer is ignored, then the temperature distribution in the laminar condensate layer is linear, and the condensation flux is specified by the balance

$$\lambda\dot m_c'' = \frac{k_L(T_s - T_w)}{\delta}; \quad \frac{d(\Gamma/\mu_L)}{dz} = \frac{k_L(T_s - T_w)}{\lambda\rho_L\nu_L\delta} \tag{5}$$

or

$$\frac{d(\Gamma/\mu_L)}{d\bar z} = \frac{N_1}{\bar\delta} \quad \text{where } N_1 = \frac{c_{pL}(T_s - T_w)}{\lambda}(\alpha_L/\nu_L). \tag{6}$$

The liquid Reynolds number, Γ/μ_L, is a function of $\bar\delta$ and the integration of equation (6) specifies $\bar\delta$ as a function of $\bar z$ and gives the solution to the problem. Here $(T_s - T_w)$ implicitly is a constant, but if this quantity is variable in $\bar z$ there is no difficulty in the integration of equation (5).

3. THE VAPOR FLOW AND THE FRICTION COEFFICIENT

No detailed consideration is made of the velocity distribution in the vapor, but rather a friction coefficient is first evaluated as for fully developed pipe flow of the vapor and then this coefficient is modified according to the specifications of Henstock and Hanratty [5], whereby there is specified an interfacial shear, in terms of a correlation of data for fully developed annular gas–liquid flow. This modification accounts for the increased friction that is due to the irregularity of the surface of the liquid layer. Fault may be found in this when it is applied to the region near the entrance of a pipe, where the friction coefficient varies substantially, but there is no similar specification for two phase annular flow and the method thus adopted underestimates the interfacial friction, τ_δ, when the distance from the inlet, z/D, is small.

The single phase friction coefficient is evaluated in terms of a vapor Reynolds number, Re_v, as

$$\frac{c_f}{2} = \frac{8}{Re_v} \quad \text{for } Re_v < 2000 \tag{7}$$

and

$$\frac{c_f}{2} = \frac{0.04}{(Re_v)^{0.25}} \quad \text{for } Re_v > 4000 \tag{8}$$

with the interpolation

$$\frac{c_f}{2} = \frac{(Re_v)^{0.33}}{3050} \quad \text{for } 2000 < Re_v < 4000. \tag{9}$$

The specification of equation (9) is not fundamental but is a device to eliminate the abrupt change in the friction coefficient that occurs at $Re_v = 2000$ if a transition to equation (8) is used at that Reynolds number. With an abrupt change in wall shear difficulties were experienced in the numerical solution of the problem of condensation in a pipe.

The Reynolds number of the vapor to be used in equation (7)–(9) should be, to be consistent with the specifications of ref. [5], $[4\,\dot m_v/(\pi D\mu_v)]$. Instead, there is used the Reynolds number $4\,\dot{m}_v/(\pi D'\mu_v)$, where $D' = D - 2\delta$, the diameter of the gas flow region. Since generally $\delta \ll D$, as it must be if the assumptions of section 2 are to be sustained, there is little difference in the choice of the Reynolds number. Actually the choice that was made is probably more appropriate, since in some instances the increase in the friction coefficient as specified by ref. [5] was not used.

For turbulent flow of the gas, ref. [5] specifies that the friction factor as given by equation (8) and (9) is to be multiplied by a quantity which will then produce the true friction factor $(c_f/2)_E$

$$\left(\frac{c_f}{2}\right)_E = \frac{c_f}{2}(1 + 1400\,F). \qquad (10)$$

For the laminar liquid flow assumed here, the quantity F is

$$F = \frac{2^{1/2}(|\Gamma/\mu_L|)^{1/2}}{Re_v^{0.9}}(\nu_L/\nu_v)(\rho_L/\rho_v)^{1/2}. \qquad (11)$$

For low vapor velocities, and particularly for downward liquid (countercurrent) flow, the factor F is to be modified further according to

$$F = F\left[1 - \exp\left(\frac{-\tau_\delta}{\rho_L g \delta}\right)\right] = F\left[1 - \exp\left(\frac{-N_2}{\bar{\delta}}\right)\right]. \qquad (12)$$

No specification such as this is available for laminar flow in the vapor region, thus for $Re_v < 2000$ the factor F is taken to be zero.

The friction coefficient as given by equation (10) applies, however, only to the case of zero velocity normal to the interface. There is no specification for the friction coefficient for the case of the substantial normal velocity due to condensation. Because of this there is adopted for the specification of the shear the sum of the above friction shear based on no normal velocity at the interface and that due to condensation at the flux $\dot{m}_c''$ (kg m^{-2} s^{-1}). This is

$$\tau_\delta = (c_f/2)_E\,\rho_v u_v^2 + \dot{m}_c''(u_v - u_{L\delta}). \qquad (13)$$

In considering this specification, note should be taken of the fact that, for the condensation problems considered, the liquid velocity at the interface, equation (3b), is less than u_v and it may be negative. Thus, equation (13) defines a 'total' friction coefficient that perhaps approximates the true value at least as $\dot{m}_c'' \to 0$ and as $\dot{m}_c'' \to \infty$. One further modification was made in the shear stress, substituting for u_v^2 in equation (13) the term $(u_v - u_{L\delta})^2$. Using the non-dimensional quantities there is then obtained

$$N_2 = (c_f/2)_E\frac{\rho_v}{\rho_L}(\bar{u}_v - \bar{u}_{L\delta})^2 + \frac{N_1}{\bar{\delta}}(\bar{u}_v - \bar{u}_{L\delta}). \qquad (14)$$

This corresponds to equation (13) for $\bar{u}_v \gg \bar{u}_{L\delta}$ and appears to be a plausible modification for use in the case in which $\bar{u}_{L\delta}$ differs considerably from $\bar{u}_v$.

With a friction coefficient specified, attention can now be given to the factor $G = [1 + (dP/dz)/(\rho_L g)]$ that occurs in equations (1)–(4). If the body force on the vapor is neglected ($\rho_v \ll \rho_L$) then a momentum balance on the vapor flow, in which the momentum rate is taken without a correction factor, as $\rho_v u_v^2$, yields for the pressure drop

$$\frac{1}{\rho_L g}\frac{dP}{dz} = -\frac{4N_2}{\bar{D}'} - \frac{\nu_v^2}{\nu_L^2}\frac{\rho_v}{\rho_L}\frac{d}{\bar{d}z}\left(\frac{4\dot{m}_v}{\pi D'\mu_G}\right). \qquad (15)$$

Since $\dot{m}_v$ decreases with distance due to the condensation, the second term on the RHS of this equation tends to compensate the effect of the first term and the group $[dP/dz)/\rho_L g)]$ is often not much different from zero. The presently indicated results for the tube neglect this term, except in the one case noted specifically in section 6. Thus, $G = 1$ in most cases though it is not too burdensome to include this term in the program whereby the evaluation of equation (6) is accomplished.

4. THE NUSSELT SOLUTION

Nusselt [1] indicated a solution for equation (6) for a constant value of N_2 and $G = 1$. The system approximating this solution can be considered as a vertical plate of infinite width and height. Vapor flows upward along the plate at constant pressure, and the friction coefficient is taken to be independent of length despite the fact that the development of a vapor boundary layer on the exterior of the condensate layer will produce a variation of the friction coefficient in the direction of vapor flow even if the second term of equation (13) is neglected. For this situation the liquid Reynolds number is, as given by equation (4a)

$$\frac{\Gamma}{\mu_L} = N_2\frac{\bar{\delta}^2}{2} - \frac{\bar{\delta}^3}{3}$$

or,

$$\frac{\Gamma}{\mu_L N_2^3} = \frac{1}{2}\left(\frac{\bar{\delta}}{N_2}\right)^2 - \frac{1}{3}\left(\frac{\bar{\delta}}{N_2}\right)^3 \qquad (4b)$$

and equation (6) becomes

$$\frac{d\bar{\delta}}{d\bar{z}} = \frac{N_1}{(N_2 - \bar{\delta})\bar{\delta}^2}$$

or,

$$\frac{d(\bar{\delta}/N_2)}{d\left(\frac{\bar{z}N_1}{N_2^4}\right)} = \frac{1}{\left(1 - \frac{\bar{\delta}}{N_2}\right)\left(\frac{\bar{\delta}}{N_2}\right)^2}. \qquad (6a)$$

This solution terminates when $N_2 = \bar{\delta}$, where $d(\Gamma/\mu_L)/d\bar{\delta} = 0$.

Figure 1 is a sketch of equation (4a) for various values of N_2. Because of the form of equation (4b), the curve for $N_2 = 1$ also represents $\Gamma/\mu_L N^3 = f(\bar{\delta}/N_2)$. For $N_2 = 1$ an expansion of the region for $\bar{\delta}/N_2 < 1.5$ is shown by an inset on Fig. 1.

For a liquid layer beginning at the bottom of the

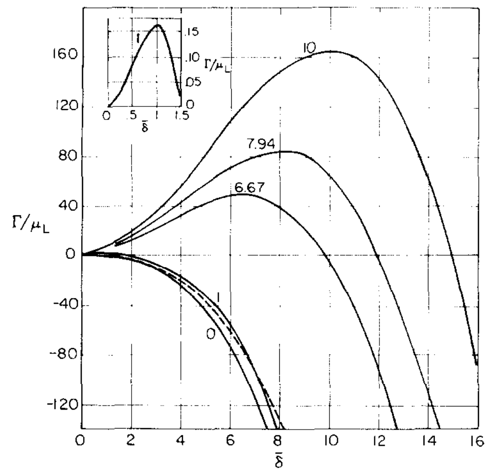

FIG. 1. Liquid Reynolds numbers as a function of non-dimensional film thickness for various values of N_2.

plate, integration of equation (6), for $\bar{\delta} = \bar{\delta}_0$ at the bottom of the plate, gives

$$\frac{4}{3}N_2(\bar{\delta}^3 - \bar{\delta}_0^3) - (\bar{\delta}^4 - \bar{\delta}_0^4) = 4N_1\bar{z} \qquad (16)$$

or

$$\frac{4}{3}\left[\left(\frac{\bar{\delta}}{N_2}\right)^3 - \left(\frac{\bar{\delta}_0}{N_2}\right)^3\right] - \left[\left(\frac{\bar{\delta}}{N_2}\right)^4 - \left(\frac{\bar{\delta}_0}{N_2}\right)^4\right] = \frac{4N_1\bar{z}}{N_2^4}.$$

If the condensate layer thickness is zero at the bottom of the plate, then upward flow of the condensate layer occurs up to the point at which the limit of $\bar{\delta}/N_2 = 1$ is attained. There the Reynolds number of the liquid is given by $\Gamma/(\mu_L N_2^3) = 1/6$, and the height at which this occurs is given by equation (16) as $4(N_1 z)/N_2^4 = 1/3$. This specifies the maximum height of a plate for upward film flow when the layer thickness at the lower edge is zero.

Completely positive Reynolds numbers can theoretically occur also if at the bottom of the plate the film thickness is such that $\bar{\delta}/N_2$ is between 1 and 3/2, so that the Reynolds number there is such as to make $0 < \Gamma/(\mu_L N_2^3) < 1/6$. This requires a supply of liquid at the bottom of the plate, a situation considered to be unrealistic in the present context. With a zero Reynolds number, $\bar{\delta}/N = 3/2$, at the bottom, half of the flow in the layer is upward and half is downward and this situation might be considered as attainable if there exists an appropriate flow reversal device at the bottom of the plate. Then the Reynolds number increases with plate height, until there is attained the limiting value of $\Gamma/(\mu_L N_2^3)$ of 1/6, when $\bar{\delta}/N_2 = 1$. Equation (16) indicates that this height is given by $4(N_1\bar{z})/N_2^4 = 0.9$. This is the maximum plate height for film flow of the condensate when a zero Reynolds number is realized at the bottom by the special means of a flow reversal device at the bottom of the plate.

If $\bar{\delta}/N_2 > 3/2$ the Reynolds numbers at the bottom of the plate are negative, and the net flow is downward. For $\bar{\delta}/N_2 = 2$ the velocity in the layer is positive throughout it, with a zero velocity at the liquid vapor interface. For $\bar{\delta}/N_2 > 2$ the velocity at the interface becomes increasingly more negative. If the Reynolds number is zero at the top of the plate, $\bar{\delta}/N_2$ must be equal to 3/2 there, but, depending upon the plate height and the vapor flow conditions, there can be positive Reynolds numbers at the top of the plate, up to the limit that is specified by $\bar{\delta}/N_2 = 1$, for which the Reynolds number is given by $\Gamma/(\mu_L N_2^3) = 1/6$. To illustrate one specific case, the one with a Reynolds number of zero at the top, and all Reynolds numbers negative and with $\bar{\delta}/N_2$ and a Reynolds number given by $\Gamma/(\mu_L N_2^3) = -1/3$ at the bottom, the height of the plate is specified by equation (16) as $4(N_1\bar{z})/N_2^4 = 4.8$.

The prior specifications, made for quantity N_2 a constant, can be modified to partly account for the contribution of the condensation to the shear as that is contained in equation (14). This involves neglecting the interface velocity, $\bar{u}_{L,\delta}$, in that equation and considering it as

$$N_2 = N_{20} + \frac{N_1\bar{u}_v}{\bar{\delta}}. \qquad (17)$$

The Reynolds number, in these terms is

$$\frac{\Gamma}{\mu_L} = \left(N_{20} + \frac{N_1\bar{u}_v}{\bar{\delta}}\right)\frac{\bar{\delta}^2}{2} - \frac{\bar{\delta}^3}{3} \qquad (18)$$

and equation (6a) can be written as

$$\frac{d\bar{\delta}}{d\bar{z}} = \frac{N_1}{\left[N_{20}\bar{\delta} + \frac{N_1\bar{u}_v}{2} - \bar{\delta}^2\right]\bar{\delta}}$$

with the solution, for $\delta = \delta_0$ at $z = 0$, the bottom of the plate

$$-\frac{4}{3}\left(\frac{N_{20}}{\bar{\delta}_0}\right)\left[1 - \left(\frac{\bar{\delta}}{\bar{\delta}_0}\right)^2\right] - \frac{N_1\bar{u}_v}{\bar{\delta}_0^2}\left[1 - \left(\frac{\bar{\delta}}{\bar{\delta}_0}\right)^2 + 1 - \left(\frac{\bar{\delta}}{\bar{\delta}_0}\right)^4\right] = \frac{4N_1\bar{z}}{\bar{\delta}_0^4} \qquad (19)$$

and with $\delta_0 = 0$ at $z = 0$ the solution is

$$\frac{4}{3}N_2\bar{\delta}^2 + N_1\bar{u}_v\bar{\delta}^2 - \bar{\delta}^4 = 4N_1\bar{z}. \qquad (20)$$

This is cited here because of later use. It cannot be considered in a general way because $4N_1\bar{z}$ now depends on both N_{20} and on $N_1 u_v$, so that the appraisal of the flow direction as made before for N_2 a constant is not possible in a concise way.

5. UPWARD VAPOR FLOW IN A TUBE

With the assumption of $\bar{\delta} \ll \bar{D}$, so that the curvature of the liquid layer can be neglected as it was in section 2, the development of the liquid layer is specified by equation (6)

$$\frac{d(\Gamma/\mu_L)}{d\bar{z}} = \frac{N_1}{\bar{\delta}}. \qquad (6)$$

In considering this equation, there can be considered the form

$$\frac{d(\Gamma/\mu_L)}{d\bar{\delta}}\frac{d\bar{\delta}}{d\bar{z}} = \frac{N_1}{\bar{\delta}} \tag{6a}$$

where

$$\frac{d(\Gamma/\mu_L)}{d\bar{\delta}} = \left(N_2 - \bar{\delta} + \frac{\bar{\delta}}{2}\frac{dN_2}{d\bar{z}}\right)\bar{\delta}.$$

It is apparent from equation (6a) that when $d(\Gamma/\mu_L)/d\bar{\delta} = 0$ this solution terminates with $d\bar{\delta}/d\bar{z} \to \infty$.

Quantity N_2 is now that given by equation (14), with $(c_f/2)_E$ evaluated as specified by equation (10). If $\bar{\delta}$ is taken to be zero initially, $\bar{\delta}$ increases with $\bar{z}$, the liquid Reynolds number is positive, and N_2 decreases with $\bar{z}$ because of the decrease in the vapor velocity. Thus, in terms of the picture of Fig. 1, the curve of the relation $\Gamma/\mu_L = f(\bar{\delta})$ originates at zero, passes through the curves for successively lower values of N_2, and terminates when $d(\Gamma/\mu_L)/d\bar{\delta} = 0$.

If $\bar{\delta}_0$ is taken to be greater than the initial value of N_2 then $\bar{\delta}$ decreases as $\bar{z}$ increases. Quantity N_2 decreases as $\bar{z}$ increases because of the decrease in the vapor velocity, and the relation $\Gamma/\mu_L = f(\bar{\delta})$ passes through the curves associated with successively lower values of N_2. It terminates at $d(\Gamma/\mu_L)/d\bar{\delta} = 0$.

This kind of behavior is shown on Fig. 1 by a dashed line, which is the last part of the solution for a specific case, case PC of section 7, which begins with a negative Reynolds number that is below the lower limit of the scale of Fig. 1. The successively lower values of N_2, due to the decreasing friction because of the decreasing value flow, are apparent. The termination, at $d(\Gamma/\mu_L)/d\bar{\delta} = 0$ cannot be discerned because of the scale of Fig. 1.

The above behavior is subject to the availability of sufficient vapor. The mass balance between the vapor and the liquid flows is

$$\dot{m}_{v0} - \dot{m}_v = \pi D(\Gamma - \Gamma_0)$$

or, in terms of Reynolds numbers

$$\frac{4}{\pi D}\frac{\dot{m}_{v0}}{\mu_v} - \frac{4}{\pi D}\frac{\dot{m}_v}{\mu_v} = 4\frac{\mu_L}{\mu_v}\left(\frac{\Gamma}{\mu_L} - \frac{\Gamma_0}{\mu_L}\right).$$

When $\dot{m}_v = 0$ the solution terminates; this may occur before $d(\Gamma/\mu_L)/d\bar{\delta}$ becomes equal to zero.

A limit on the inlet gas Reynolds number for a given tube size and fluid properties can be established for the case of complete condensation, as would occur in an infinitely long tube. Regardless of the nature of such condensation, all of the liquid must then leave the bottom of the tube. If that condensation is completely film-wise, this requires that all of the liquid Reynolds numbers along the tube height be negative, terminating with $\delta/N = 3/2$ and $d(\Gamma/\mu)/d\bar{\delta} = 0$ simultaneously at the point where all of the vapor is condensed, so that also, N_2 would be zero at that point. This has not been proven and this condition may not always exist. If it does not, the condensation must be completed by means other than exclusively film-wise, but in any case all of the liquid must leave at the bottom of the tube.

If the condition $\bar{\delta}/N_2 = 2$ is adopted for the bottom of the tube with complete condensation, then for this value of $\bar{\delta}/N_2$ the liquid Reynolds number is $(-2\,N_2^3/3)$. A choice of a vapor Reynolds number, for a given tube diameter and given properties, specifies N_2 and thus the liquid Reynolds number. But the mass flow of liquid so specified by the liquid Reynolds number must equal the mass flow of vapor associated with the chosen gas Reynolds number, as from equation (21)

$$\frac{4}{\pi D}\frac{\dot{m}_{v0}}{\mu_v} = -4\frac{\mu_L}{\mu_v}\left(\frac{\Gamma_0}{\mu_L}\right)$$

A trial and error solution made in an approximate way by neglecting the second term in equation (14), establishes the gas Reynolds number that meets these conditions. It is the maximum gas Reynolds number for complete condensation. Lower inlet gas Reynolds numbers than this one will produce lower values of N_2 at the tube inlet, and liquid films leaving the tube with larger values of $\bar{\delta}/N_2$. This maximum Reynolds number is cited in some of the evaluations that follow.

6. THE CALCULATION AND RESULTS

Given the initial value of $\bar{\delta}$ and the initial vapor flow, which specifies the initial value of $[4\,m_v/(\pi D\mu_v)]$, the solution of equation (6) gives $\bar{\delta}$ as a function of $\bar{z}$. This solution is made in the following way:

(1) The initial value of $\bar{\delta}_0$, together with the initial value of m_{v0}, enables the calculation of an initial value of N_2 from equation (14) and the associated definitions of the friction factor.

(2) A suitably small increment in $\bar{\delta}$ is selected, to define the next value of $\bar{\delta}$, which is $\bar{\delta}_1$.

(3) This defines an approximate Γ/μ_L by the use of the initial value of N_2. The mass balance gives a value of the gas Reynolds number, $[4\,m_v/(\pi D\mu_v)]$ and then a new value of N_2 is calculated. Linear interpolation is used to find, from the initial and the new values of N_2, a more suitable value of N_2, which is then used to find another value of Γ/μ_L and the process is repeated until the values of Γ/μ_L are within 0.3% of each other. The liquid Reynolds number is then taken as the value given at the end of this iteration.

(4) The increment in $\bar{z}$ is calculated from

$$N_1\Delta\bar{z} = \left(\frac{\bar{\delta}_0 + \bar{\delta}_1}{2}\right)\Delta\left(\frac{\Gamma}{\mu_L}\right).$$

(5) With (Γ/μ_L), $\bar{\delta}$, N_2 now specified, another increment is taken in $\bar{\delta}$, as in step 2, and the process is repeated. The calculation is carried out either to the point at which $d(\Gamma/\mu_L)/d\bar{z} = 0$ or to the point at which all of the vapor has been condensed.

If $\bar{\delta}_0 = 0$, gravity is neglected for the first step, so that the $\bar{\delta}^3$ term does not appear in the denominator of equation (6a) and then, with N_2 taken from equation (14), with F taken to be zero in equation (10), equation

Table 1. Prediction summary. Cases A, properties at 100°C, Pressure 1.0 atm; $(T_s - T_w) = 55°C$; $N_1 = 0.0581$; $D = 1.27$ cm; $\bar{D} = 607$. P, properties at ≈260°C, Pressure 61.2 atm $T_s - T_w = 39°C$; $N_1 = 0.157$; $D = 1.97$ cm; $\bar{D} = 1620$

Column	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
		Initial conditions						Final conditions					
Case	δ_0	$\frac{4\dot{m}_v}{\pi D \mu_v}$	$\frac{\Gamma}{\mu_L}$	N_{20}	$N_1 \bar{u}_v$	N_2	δ	$\frac{4\dot{m}_v}{\pi D \mu_v}$	$\frac{\Gamma}{\mu_L}$	$\bar{z}$	$\frac{z}{D}$	% cond.	1400*F*
AA	0	25600	0	20	185.5	6.6×10^4	10.6	4830	236	9340	15.4	82	0.162/8.26
AB	0	1890	0	0.123	13.5	94	2.20	1290	6.5	78	0.13	31	0
AC	5.40	1890	−13.1			2.7	1.42	564	1.8	1050	1.74	70	0
AD	5.60	1890	−17.4	0.147	13.8	2.6	1.00	289	0.67	1279	2.11	85	0
AE	5.75	1890	−20.9			2.6	0.58	39	0.01	1471	2.42	98	0
PA	0	7865	0	0.08	3.16	45.7	1.29	7850	1.1	3.5	2×10^{-3}	0.3	0.121/0.038
PB	11.08	7865	−352	(0.80)		1.6	0.02	3.9	≈ 0.04	18354	11.3	99.98	0
PC	11.69	7865		(2.2)		2.6	0.01	2.3	≈ 0.04	18875	11.7	99.99	1.95/0

(6a) can be integrated analytically. Thus, δ is determined as a function of z. A very small value of δ is chosen. Then a new value of N_2 is calculated and these values serve for the entrance to step 2, as above.

Calculations were carried out in this way for two different situations as defined at the top of Table 1. One was for water at atmospheric pressure, with properties of both liquid and vapor evaluated for saturation conditions. The temperature difference between the vapor and the wall was taken as 55°C. Since the liquid temperature is therefore lower than the saturation temperature, the liquid property evaluation is inconsistent, but the situation is tolerable since the results are merely an example. As noted before, if the liquid properties are considered to be invariable with temperature, then the distances predicted for the chosen temperature difference apply for any other temperature difference if the distance is taken to be inversely proportional to the temperature difference. All of the cases for atmospheric pressure are designated by letter A.

The second set of cases, designated by letter P, involve water at a pressure of 900 psia, with a tube diameter of 1.97 cm, slightly greater than that used in the A cases. In the P cases the vapor properties were taken at saturation conditions and the liquid properties at a film temperature corresponding to the chosen difference of 39°C between the vapor and the wall temperature. The P cases are related to a specification of conditions at entrance to the steam generator of the TMI system at a particular time in the TMI event. This is the reason for the choice of the conditions for the P cases.

The entries of Table 1 define the cases by two letters, the first as indicated above and the second which merely defines the particular cases and has no other significance. The initial conditions as assumed for a particular case are specified in Columns 1 and 2 of the table. The assumed value of $\bar{\delta}_0$, Column 1, is zero for some cases and finite for others. The assumed initial value of the vapor Reynolds number based on tube diameter, Column 2, reflects as assumption of an initial steam rate, $\dot{m}_v$, kg s^{-1}. These two assumptions, together with a definition of the friction coefficient, specify the initial value of N_2 via equation (14), as given in Column 6. This specification of the friction coefficient involves the quantity, F, equation (10). As noted in that connection F was taken to be zero if the vapor Reynolds number was less than 2000. For a number of cases the vapor Reynolds number is less than this initially, and this fact is noted by a zero value for F in Column 13. In one other case, PB, it was arbitrarily taken to be zero and this is noted in Column 13. For cases AA, PA and PC, Column 13 shows the initial and final values of the quantity 1400F.

With $\bar{\delta}_0$ and N_2 specified, the liquid Reynolds number is specified as in Column 3; negative values indicate that Γ, the liquid flow rate per unit width, is negative. Columns 4 and 5 are not fundamental, and are associated with certain approximate predictions made for the Nusselt solution of Section 4, done to provide some orientation for the basic results. Entries appear only for the cases in which such a presentation is made. Column 4 gives N_{20}, the value associated with equation (19) calculated from the vapor flow condition assuming that $u_{L\delta}$ is zero. Column 5 gives $N_1\bar{u}_v$, associated with the contribution of the condensation to the shear stress, neglecting the effect thereon of $\bar{u}_L$.

The remaining columns of the Table refer to the final conditions of the calculation. All terminated for $d(\Gamma/\mu_L)/d\bar{\delta} = 0$, though for some of them this termination was very close to the other termination point that would have been executed upon complete condensation of the available steam. Column 7 gives the final value of $\bar{\delta}$, Column 8 the final vapor Reynolds number, Column 9 the final value of the liquid Reynolds number, and Column 10 the final value of $\bar{z}$. Column 11 gives z/D, the number of tube diameters from the inlet at which the final $\bar{z}$ occurs. Column 12 gives the percentage of the inlet steam flow that is condensed.

The values of $\bar{\delta}$ and of Γ/μ_L that were calculated between the initial and final points are presented as functions of $\bar{z}$ on figures.

7. RESULTS FOR STEAM CONDENSATION AT ATMOSPHERIC PRESSURE

For atmospheric conditions, it was noted previously that an approximation could be made for the inlet vapor Reynolds number that would result in complete condensation, with all of the condensate running out of the bottom of the tube. For the conditions of the A cases this Reynolds number is 4200. Table 1 indicates case AA for an initial vapor Reynolds number of 25 600, so much higher than 4200 that an initial liquid layer thickness of zero is the only reasonable assumption. This calculation terminates with $d(\Gamma/\mu_L)/d\bar{\delta} = 0$ at $\bar{z} = 9340$ ($z/D = 15.4$), with 81.6% of the vapor condensed and a positive liquid Reynolds number of 256. If the tube is only 15.4 diameters long for the 55°C temperature difference or in inverse proportion to the temperature difference for other temperature differences, this is the outlet condition at the top of the tube. If the tube is longer than this, the condensation will continue, but with a different flow regime than the annular flow that is assumed in the model. Even at termination δ/D is only 10.6/620; thus the liquid volume fraction is relatively small. One hypothesis, not yet examined, would be a continuation of the annular flow at the limiting value of δ with entrainment of the excess condensation in the vapor stream.

Figure 2 indicates the variation of $\bar{\delta}$ with $\bar{z}$ for this case in terms of $\bar{\delta}$ as a function of $4N_1\bar{z}$. As a point of reference, the Nusselt solution as given by equation (20) is shown as a dashed line for the initial values of N_{20} and $N_1\bar{u}_v$ as they are given in Table 1. The more rapid attainment of the terminal condition in the pipe flow case, due to the decrease in the vapor velocity, is evident clearly. Figure 2, on another curve, shows the variation of the liquid Reynolds number, Γ/μ_L, for the solution for the pipe. There is shown no comparison of this to the Reynolds number associated with the Nusselt solution.

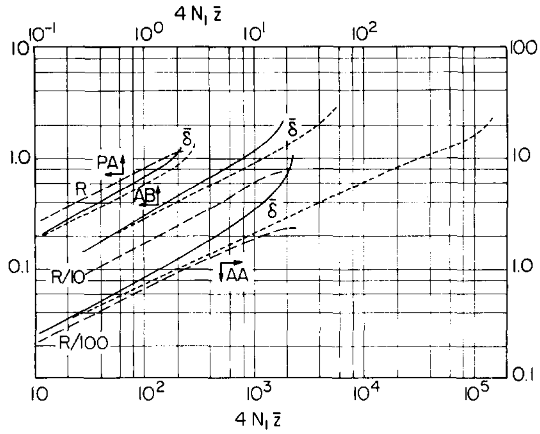

FIG. 2. The variation of $\bar{\delta}$ and Liquid Reynolds number along the height of a vertical tube. Cases AA, AB, and PA are indicated, with ordinate and abscissa scales as specified for them. The solid curve for $\bar{\delta}$ is the computed result; the dashed curve is an approximation for the condensation on a vertical plate with constant vapor velocity at the initial value thereof. Curves R are the liquid Reynolds number, Γ/μ_L.

A case like that of Case AA was calculated, including the value of G that is defined by equations (3) and (15). The effect on $\bar{z}$ for the termination point, of $d(\Gamma/\mu)/d\delta = 0$, was very small. This does not support, of course, the use of $G = 1$ in all of the cases given in Table 1, and in the future the value of G should probably be included at the expense of additional complication in the part 3 of the calculation as outlined before.

Another Case, AB, is presented for an initial vapor Reynolds number of 1890. This is far less than 4200 and downward liquid flow is indicated. Nevertheless, if $\bar{\delta}$ is taken to be zero at the inlet, the terminal condition is indicated to be at $\bar{z} = 70.7$, $z/D = 0.13$, with 31% of the vapor condensed. If the tube was indeed only 0.13 diameters high, this situation could in principle be attained. Figure 2 shows $\bar{\delta}$ as a function of $4\bar{z}N_1$ with different scales being used because of the small values that are involved in this case. A dashed line shows the solution from equation (20). Despite the very short length there is considerable condensation and the value of $\bar{\delta}$ departs considerably from this Nusselt solution even in the short distance that is involved.

For this initial Reynolds number of 1890, three cases are shown for arbitrary but different values of $\bar{\delta}_0$, Cases AC, AD, and AE, involving successively lower values of the negative liquid Reynolds number at the tube inlet, and for these cases the condensation ranges from 70 to 98% and for the latter only 2.42 tube diameters of height are required. The 9% is so close to the 100% level that any tube of length greater than about 2.5 diameters will condense all of the stem input that was specified for these cases. Figure 3 shows by points the relation of $\bar{\delta}$ to $\bar{z}$, given there as (δ/δ_0) as a function of $(4(N_1\bar{z})/\bar{\delta}_0^4)$. There is little distinction between the results except for the termination point, indicated by the last, underlined, symbol for each case. For a

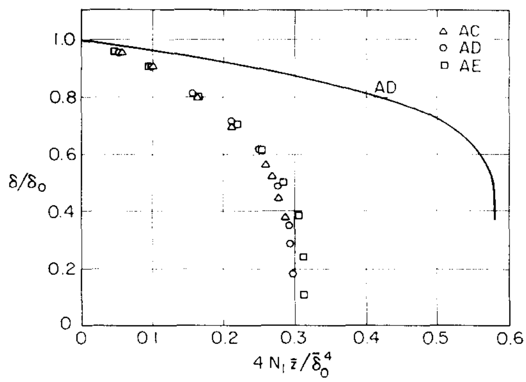

FIG. 3. The variation of $\bar{\delta}$ with height, Cases AC, AD, and AE. The points are derived from the computed results. The curve approximates the flat plate result for Case AD.

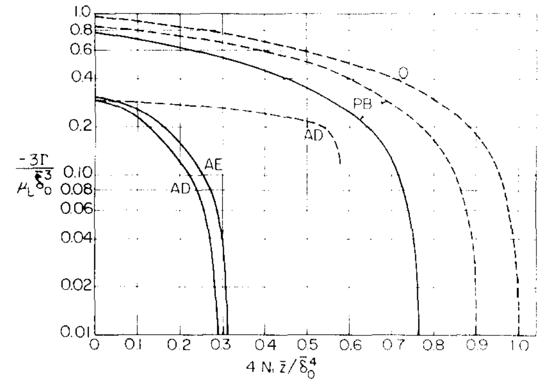

FIG. 5. The variation of liquid Reynolds number with height. The solid curves show the computed results for Cases AD, AE, and PB. All Reynolds numbers are negative except for the terminal values for Case AD, barely discernible as an almost vertical line near the ordinate of 0.3. These final Reynolds numbers are positive. The dashed curves for Cases AD and PB are the Reynolds numbers for the plate predictions that are shown on Figs. 3 and 4.

comparison, equation (19), the Nusselt solution for the condition of Case AD, is shown by a curve on the figure. The variation of the Reynolds number with height is shown on Fig. 5 by curves for Cases AD and AE, and the prediction for Case AD of the Reynolds number as given by equation (18) is shown by a dashed curve on the figure. This figure also contains the curve, 0, representing the Nusselt solution for a vertical plate for the case of stationary vapor.

8. RESULTS FOR STEAM CONDENSATION AT 900 psia

As noted in section 5, complete condensation, provided that the tube was high enough, with downward flow of the condensate, would be expected for gas Reynolds numbers up to about 50 000 for the 1.29 cm diameter tube under consideration for the high pressure condition, for which the P cases defined in Table 1 were calculated. All of those are for an initial vapor Reynolds number of 7865, chosen for the specific problem of the TMI stream generating unit. Thus downward condensate flow is expected and Cases PB and PC deal with such a situation. As a matter of interest, however, a calculation, Case PA, was made for this initial vapor Reynolds number and an initial liquid layer thickness of zero, so that the liquid flows upward in the tube.

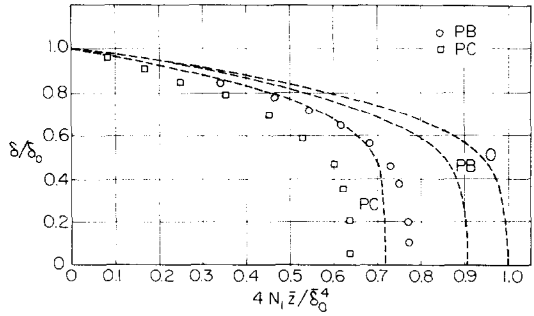

FIG. 4. The variation of $\bar{\delta}$ with height, Cases PB and PC. The points are derived from the computed results. The curves are approximation for the vertical plate case, qualified as in section 7.

For Case PA, Table 1 shows a termination, with $d(\Gamma/\mu_L)/d\bar{\delta} = 0$, at $\bar{z} = 3.5$, corresponding to a distance of only 2×10^{-3} diameters from the inlet. Only 0.3% of the stream flow is condensed before the liquid layer becomes thick enough so that the upward condensate flow can no longer be maintained. The dependence of $\bar{\delta}$ on $\bar{z}$ is shown on Fig. 2 by a solid curve. The dashed, comparison curve from equation (20) is very close to the computed solution because very little of the vapor is condensed. A solid curve, R, on Fig. 2 shows the variation of the Reynolds number.

Table 1 indicates two cases, PB and PC, for different but similar values of $\bar{\delta}_0$. In case PB, the factor for friction augmentation, F, was arbitrarily taken to be equal to zero, while it was used as specified in section 3 for Case PC. It is the difference in N_2 associated with this choice that causes the initial Reynolds number for the liquid to be the same for these two cases. The final values that are indicated in Table 1 show that for the chosen initial conditions the effect of the factor F is not very important. Both of these cases show essentially complete condensation, though the termination was at $d(\Gamma/\mu_L)/d\bar{\delta} = 0$. (Case PC is the one for which the latter part of the relationship between Γ/μ_L and δ is shown by a dashed line on Fig. 1.) The liquid Reynolds number at the termination was positive but only very slightly greater than zero. Case PB terminated at 11.3 tube diameters from the inlet and case PC at 11.7 tube diameters. A tube of this length, or any longer tube, will produce complete condensation of the inlet steam flow, with all of the condensate emerging from the bottom of the tube.

Figure 4 shows by points the variation of (δ/δ_0) with respect to $(4\bar{z}N_1)/\bar{\delta}_0^4$ for Cases PB and PC. These points do not imply the increments that were used in the calculations, but are only selected from the results. The calculations, for Case PC for example, involved 139 increments in $\bar{\delta}$. The Nusselt solution for a vertical plate is indicated here by dashed curves for the two cases, as obtained from equation (17) of section 4, using the value of N_{20} indicated in Column 4 of Table 1. Again, as for the somewhat similar but more accurate specification for Case AD on Fig. 3, there is an indication of the more rapid termination of the condensation in the tube because of the reduction of the vapor velocity that takes place in the tube.

Figure 5 contains a curve for Case PB to show the variation of the liquid Reynolds number with height and shows by a dashed curve the variation of the Reynolds number associated with the flat plate case, associated with the result from equation (17) as that is shown on Fig. 4.

The applicability of these theoretical results, for cases like PB and PC, and also Case AE, is in question in any actual situation in respect to the possibility of the downward drainage of the condensate from the bottom of the tube without interference with the incoming steam. In an apparatus that might be contemplated to check experimentally those computed results, a collection region for the condensate would need to be of such form that the supply steam could enter the tube without encountering the condensate stream. An annular trough for condensate collection might achieve this.

9. SUMMARY

The Nusselt solution for condensation from the upward vapor flow over the surface of a vertical plate has been reviewed as an introduction to the case of condensation from an upward vapor flow in a circular tube. Some comments have been made about the possible directions of the condensate flow but a definition of a specific condensate flow regime does not emerge from this consideration.

For condensation of steam in tube, computations have been made for specific inlet conditions and examples of these are used to indicate the success of the computational procedure and to demonstrate cases of upward and downward liquid flow. Some of these cases are for a pressure of 900 psia, and are associated with a tube size and inlet flow defined for an operating condition of the TMI steam generating unit. They show complete condensation in a very short height of the tube, with condensate flow out of the bottom of the tube. There remains, however, the question about the ultimate course of this condensate, and whether it will intermingle in part with the incoming steam.

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CONDENSATION EN FILM LAMINAIRE DANS UN TUBE AVEC ECOULEMENT ASCENDANT DE VAPEUR

Résumé—La solution de Nusselt pour la condensation en film laminaire sur un plan vertical avec écoulement ascendant de vapeur est reprise avec des commentaires sur la nature de l'écoulement de condensat. Le même problème est considéré dans le cas de l'intérieur d'un tube vertical pour lequel la décroissance du débit de vapeur du fait de la condensation produit une réduction du cisaillement qui conditionne la couche de condensat. Une solution numérique du problème est alors requise et des résultats sont donnés pour illustrer la méthode de calcul et pour indiquer la nature du processus de condensation dans le tube vertical. Ces cas concernent à la fois l'écoulement ascendant et descendant de condensat.

LAMINARE FILMKONDENSATION IN EINEM ROHR BEI AUFWÄRTSGERICHTETER DAMPFSTRÖMUNG

Zusammenfassung—Die Nusselt'sche Lösung für die laminare Filmkondensation an einer senkrechten Platte mit aufwärtsgerichteter Dampfströmung wird mit Anmerkungen zur Art der Kondensatströmung behandelt. Das gleiche Problem der laminaren Filmkondensation wird für den Fall einer aufwärtsgerichteten Dampfströmung in einem senkrechten Rohr betrachtet. Dabei führt das Abnehmen des Dampfmassenstromes durch die Kondensation zu einer Verringerung der Schubspanning, die auf die auf die Kondensatschicht einwirkt. Das Problem erfordert eine numerische Lösung. Für einige Spezialfälle werden Ergebnisse mitgeteilt, um das Rechenverfahren zu erläutern und die Natur des Kondensationsvorganges in einem senkrechten Rohr zu veranschaulichen. Diese Fälle umnfassen sowohl die nach oben als auch die nach unten gerichtete Kondensatströmung.

ЛАМИНАРНАЯ ПЛЕНОЧНАЯ КОНДЕНСАЦИЯ В ТРУБЕ С ВОСХОДЯЩИМ ПОТОКОМ ПАРА

Аннотация—Рассматривается решение Нуссельта для ламинарной пленочной конденсации на вертикальной пластине при восходящем потоке пара. Особое внимание обращено на выяснение природы потока конденсата. Та же проблема ламинарной пленочной конденсации рассматривается для случая восходящего потока пара на внутренней стороне вертикальной трубы, когда снижение скорости расхода пара из-за конденсации вызывает изменение касательного напряжения, удерживающего слой конденсата. Выполнено численное решение задачи с целью иллюстрации метода расчета и объяснения природы процесса конденсации в вертикальной трубе и приведены результаты для частных случаев. Рассмотрены как восходящее, так и нисходящее течения конденсата.